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Unified Equation Formulation for Electronic and Electrical Circuits Analysis

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Abstract - In an attempt to bring together simulation tools and particularly equation formulation methods of power electrical systems analysis and electronic circuit analysis we were studying and contrasted to each other the properties of Load Flow Analysis (LFA) and the Modified Nodal Analysis (MNA). Based on comparisons of their implementations presented in textbooks we came to the conclusion that MNA is favourable as compared to LFA from several points of view such as comprehension, simplicity and universality.

Keywords - Circuit analysis, Power systems simulation.

I. INTRODUCTION

After Tesla's introduction of alternating current and poly-phase systems at the end of the nineteenth and the beginning of the twentieth century, electrical engineering and electricity as such became and still is one of the most important vehicles of the development of human society. However, over time the discipline diverged in two directions: power electrical systems and electronics. For a long time, these separate research communities operated in independent silos. It was only recently that electronics and ICT became necessary for the modernization of power production, distribution, and consumption which has become known as the smart grid. On the other hand, it was only recently that electronics and ICT became one of the biggest consumers of electricity, and as a secondary consequence through the necessary power converters, begun to seriously threaten the quality of the delivered power [1].

Similar separation and re-convergence was to befall the design tools developed for these two trades, including simulation software. Today, modern power electrical system design literature covers subjects that are also electronics oriented [2]. However, simulation tools being developed as part of the power system design subsystems are, of course, not able to reproduce electronic components down to transistor level. On the other hand, simulation tools developed as part of electronic and ICT system design are not able to reproduce various phenomena specific to power generation, transmission and distribution systems. Given the demands that modelling the modern smart grid places on both domains, researchers should extend the capabilities of existing tools such that both worlds are covered [2]. This paper is an attempt to help achieving this goal.

In fact, this is an effort to go deeper into the subject of simulation by facing one against the other the equation formulation methods underlying the simulation programs coming from the two domains. In that way, the intention is to promote better mutual understanding between communities and facilitate convergence toward an omnipotent simulation tool.

As a vehicle of the comparison and the proposal, the examples from two modern text books of power electrical and smart grid systems are used. After repeating part of the example is given in one of them, its deficiencies are exposed and a solution, which is not only successful and effective but also easy to understand and to teach new generations, is proposed.

II. LOAD FLOW ANALYSIS

In the book [3] while introducing the theory of Load Flow Analysis (LFA), on page 396, the following "Example 7.2" is given (a similar, but conceptually identical, example is given in [4] on page 213):

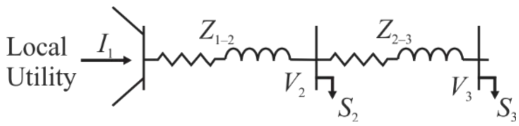

Fig. 1. Distribution feeder (picture 7.2 in [4])

"Example 7.2: Consider a distributed feeder presented in Fig. 7.2. Assume the following:

a. Feeder line impedances, that is Z_{1-2} and Z_{2-3} are known.

b. The active and reactive power consumed, that is S_2 and S_3, by loads are known.

c. The local power grid bus voltage V_1 is known and all data are in per unit."

A solution follows, which we have to repeat due to its

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fundamental importance to the knowledge delivered to the power electrical engineering community. Namely, it seems that the way of thinking expressed in this example is omnipresent and, in our opinion, difficult to fit into the fundamental laws of electrical circuit analysis. It is as follows:

"Solution: Let us write the Kirchhoff's current law for each node (bus) of Fig. 7.2 and assume that the sum of the currents away from the bus is equal to zero. That is, for the buses 1-3, we have

$$\begin{aligned} (v_1 - v_2)\, y_{12} - I_1 &= 0 \\ (v_1 - v_2)\, y_{12} + (v_2 - v_3)\, y_{23} + I_2 &= 0 \\ (v_3 - v_4)\, y_{23} + I_3 &= 0 \end{aligned} \tag{1}$$

where $y_{12} = 1/Z_{1-2}$, $y_{23} = 1/Z_{2-3}$, and

$$I_1 = \left(\frac{S_1}{V_1}\right)^*, \quad I_2 = \left(\frac{S_2}{V_2}\right)^*, \quad I_3 = \left(\frac{S_3}{V_3}\right)^* \qquad (2)."$$

After substitution, the author comes forward with the following:

"The above can be written as

$$\begin{bmatrix} Y_{11} & Y_{12} & 0 \\ Y_{21} & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}, \tag{3}$$

where $Y_{11} = y_{12}$, $Y_{12} = -y_{12}$, $Y_{21} = -y_{12}$, $Y_{22} = y_{12} + y_{23}$, $Y_{23} = -y_{23}$, $Y_{32} = -y_{32}$ and $Y_{33} = y_{23}$. The matrix equation (3) represents the bus admittance matrix; it is also the Y_{Bus} model for Example 7.2."

Equation (3) in fact is not a final solution. It can't be, and we will come back to that. To come to the final, the authors in [3] and [4] turn from voltages and currents into power by creating nonlinear equations describing a linear system. These, naturally, need special algorithms in order to be solved (Gauss-Seidel is advised first). That includes creating initial solutions and control of convergence. In [5] Newton-Raphson is further recommended due to its faster convergence.

III. Modified Nodal Analysis

Nodal analysis (NA) is based on the Kirchhoff's current law and as it is already explained above the node equations is expressing the following: The sum of currents leaving the node is equal to zero. The node voltages are unknown while the currents are first expressed as functions of node voltages using the constitutive equations of the circuit elements connected to a node. Frequency domain analysis is facing a problem when ideal voltage source is connected between nodes since the branch current cannot be expressed as a function of the branch voltage (ideal voltage source has no Norton equivalent). Time domain analysis, in addition, has problem to express the voltage equation of the inductance. Both problems are solved when Modified Nodal Analysis (MNA) is implemented [6,7]. The trick is that the branch current of the voltage element (ideal voltage source or inductor) is introduced as a new variable enabling the proper node equations to be assembled. In that way the number of variables is incremented by 1. In addition, the system of circuit equations is extended by a new equation related to the branch of the voltage element. Fig. 1 depicts an ideal voltage source connected between nodes *j* and *k*, and the "stamp" or the contribution of such an element to the system of equations describing the circuit.

Implementation of MNA is not domain dependent. This means that one uses the same concept for (power-) frequency and the time domain analysis [8], while the system to be analyzed may be linear or nonlinear. To our knowledge there are serious attempts within the power electrical engineering community to adopt MNA and implement it for system simulation [5,8-13]. That is not the case for the LFA as it is thought at universities and hence our intervention.

Node	v_j	v_k	i_{new}
j			1
k			-1
New equation	1	-1	E

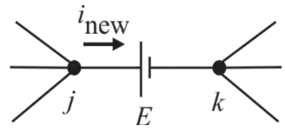

Fig. 2. An ideal voltage source connected between nodes *j* and *k*, and the "stamp" or the contribution of such an element to the system of equations describing the circuit.

IV. Load Flow Analysis Versus Modified Nodal Analysis

We will start here to express our full disagreement with developments expressed in second section. To begin with, the node voltage V_1 is already known and one should not expect that the solution of (3) would produce the same value as the original. Then, node 1 (bus bar 1) has no internal impedance which is easily recognized from the fact that S_1 is missing in Fig. 2. That may look as unimportant mistake but in fact is a fundamental error misleading the whole analysis. As already mentioned ideal voltage source has no Norton equivalent meaning that I_1 cannot be related to V_1. The bus bar current is unknown until the end of the analysis since it depends on the load. Theoretically, for the same bus bar voltage, it may vary from zero to infinity. If, however, the ideal voltage source was to be kept in the circuit it would be impossible to formulate the nodal equations as above. One would need to use MNA by which I_1 would be introduced as unknown circuit variable and the order of the system of equations would be raised by 1.

Further, equation (2) is also misleading. Namely, a set of unknown variables (V_i, $i = 1,\ 2,\ 3$) is used to create a set of other unknown variables (E_i, $i = 1,\ 2,\ 3$) while both stay in the same system. According to the author, to get the "Y_{Bus} model" one should create a system of three equations with six unknowns. That is difficult to believe, and as already commented it leads nowhere. Furthermore, if the node voltages change why would the load power stay constant? Namely, the truth is $\mathbf{S}$, as given in the figure, are *nominal powers* which, for the given impedances are valid only for the *nominal voltages*. The load impedance is the only invariant (since steady load is presumed). In our opinion the loads were to be noted as nominal ($\mathbf{S}_N$) being valid only for nominal voltages ($\mathbf{V}_N$) and used to find the admittances only.

To put the same comment in other words, if $\mathbf{I}$ is unknown (since $\mathbf{V}$ is unknown), why are they on the right-hand side of the equation? Or, how many complex valued unknowns are in (2) and (3), three, six or nine?

Unfortunately, the story does not end here. Namely, looking at the final "Y_{Bus} model" and switching off the source (putting $I_1 = 0$) one gets

$$\begin{bmatrix} Y_{11} & Y_{12} & 0 \\ Y_{21} & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ I_2 \\ I_3 \end{bmatrix} \tag{4}$$

or

$$\mathbf{Y} \cdot \mathbf{V} = \begin{bmatrix} 0 \\ I_2 \\ I_3 \end{bmatrix}, \tag{5}$$

whose solution is

$$\mathbf{V} = \mathbf{Y}^{-1} \cdot \begin{bmatrix} 0 \\ I_2 \\ I_3 \end{bmatrix} \neq \mathbf{0}. \tag{6}$$

Can the grid produce node voltages even if there is no power source active in it?

Here we come to one of the fundamental mistakes done when loads are represented in LFA (and not only in it). Namely, instead of impedances (which are in fact the loads) one uses current sources. However, a current source is a *source of energy* even if it is negative by value while the impedance is a *consumer of energy*. Interchanging those leads to wrong solutions (like, having voltages and currents in absence of real sources in the system). Furthermore, looking in circuit theoretical mode, the impedance's current changes linearly with the change of its voltage. The current of a current source is constant and independent of the voltages drop on it (looking at it as if it is impedance, it becomes nonlinear conductance with hyperbolic decay). Finally, by use of a constant current source we ignore any change of the currents in all loads. So, since we know the currents ($\mathbf{I}$) and the loads ($\mathbf{S}$), there is no need for any analysis. The voltages may be obtained from (2) backwards.

To conclude, both node voltages and load currents (and consequently load powers) are unknown at the beginning of the analysis and the concept expressed in [3,4] is misleading.

One is not to forget that the books we are speaking about ([3] had two and [4] eight editions) are intended to be read by novices (students).

Before proceed to implementation of the MNA as an ultimate solution of the equation formulation problem we will try to accommodate to the NA. Of course, we will suppose that $\mathbf{V}_N$ and $\mathbf{S}_N$ are known which means we know the load admittances Y_{L2} and Y_{L3}.

Supposing I_1 is known, one is to substitute the admittances of the loads and proceed with the formulation in the form

$$\begin{bmatrix} Y_{11} & Y_{12} & 0 \\ Y_{21} & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 0 \end{bmatrix}, \tag{7}$$

where $Y_{11} = y_{12}$, $Y_{12} = Y_{21} = -y_{12}$, $Y_{22} = y_{12} + y_{23} + Y_{L2}$, $Y_{23} = -y_{23}$, $Y_{32} = -y_{32}$ and $Y_{33} = y_{23} + Y_{L3}$.

Again, Y_{L2} and Y_{L3} are the admittances of the loads which are passive and do not generate energy (hence, must be on the left-hand side of the equation).

Since V_1 (not I_1) is known, by simple manipulations one gets

$$\begin{bmatrix} -1 & Y_{12} & 0 \\ 0 & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -Y_{11}V_1 \\ -Y_{21}V_1 \\ 0 \end{bmatrix}. \tag{8}$$

This seems an acceptable final solution. It resembles the case when the admittance of the first line, $Y_{11} = 1/Z_{1-2}$, is used as internal admittance of the voltage source V_1. This (handmade) trick is, unfortunately, not applicable when more than one line is connected to the bus bar.

In other words, we would prefer a universal method enabling in the same time for automatic (computer generated) equation formulation. Here comes the MNA. If MNA was to be applied instead (7) one would have:

$$\begin{bmatrix} Y_{11} & Y_{12} & 0 & -1 \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ 0 & Y_{32} & Y_{33} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E_g \end{bmatrix} \tag{9}$$

Here a voltage source (E_g) is connected to node 1. This system complies with the equation formulation used in the SPICE program [12,14] meaning that if SPICE description were created one would produce the solution in no time. Note there are versions of SPICE available for free.

To finalize, one is not to forget that when all node (or branch) voltages are known we may calculate *everything*: currents and powers, meaning there is no need to switch to powers before the node voltages are found.

V. CONCLUSION

To conclude, it is our opinion that it is time to demystify the power system analysis (including PFA) as taught until now, since despite of a very long period of use of such systems is still obscure and ineffective. Using a simple, but as general as necessary, example we showed that MNA should be taught to our student as a universal means of equation formulation for circuits and systems simulation.

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